layer In using the terms "slip flow" and "rarefaction effects," the writer had in mind the combination of all of these phenomena, and for this reason it was not unreasonable to suppose that these effects might correlate better with a mean free path behind the shock wave rather than at the wall Whether or not these "rarefaction phenomena," in the sense that the writer used the term, are susceptible to analysis within the framework of Navier-Stokes theory is still an open question It is his opinion, evidently shared by Aroesty, that the resources of the Navier-Stokes equations are not yet exhausted

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Flow in Contracting Ducts

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THERE are a few points of interest arising from the paper by Szczeniowski [1] He draws attention to two solutions for the two-dimensional irrotational, incompressible flow through contracting ducts These are, of course, two of many such existing solutions, some of which have been critically compared by Moretti [2]

It is of interest to examine further the second solution quoted by Szczeniowski [1] The expression for the ordinates in the $z \equiv x + iy$ plane in terms of the complex potential $w \equiv \varphi + i\psi$ is given as

$$e^{\ /a} = e^{\mu(w/aU)} + e^{w/aU}$$

Taking, for algebraic simplicity, both a and U as unity, differentiating and rearranging results in

$$w = \frac{1}{\mu - 1} \log\left(\frac{dw}{dz} - 1\right) - \frac{1}{\mu - 1} \log\left(\frac{dw}{dz} - \frac{1}{\mu}\right) + \frac{i\pi - \log\mu}{\mu - 1}$$

where $i\pi$ is taken as the principle value of $\log(-1)$ Now the dw/dz plane is the hodograph plane, for $dw/dz = u - iv$ Thus, in this plane the flow is from a source of strength $1/(\mu - 1)$ at the point (1,0) to a sink of the same strength at the point $(1/\mu,0)$ This solution has also been described by Libby and Reiss[3] and the present writer [4] The outermost streamline along which the velocity rises monotonically is the circle whose diameter lies along the x axis Along the axis corresponding to the duct centerline, $1/\mu \leq dw/dz \leq 1$ (note that the flow is backward in the duct), and so $+\infty \geq \varphi \geq -\infty$ and $\psi = 2\pi/(\mu - 1)$ The outermost streamline has a stream function value that is less than this by $(\pi/2)/(\mu - 1)$ and thus, along it, $\psi = (3\pi/2)/(\mu - 1)$ for all values of μ ($\mu > 1$)

It does not appear possible to obtain any exceptions to the result that the velocity distribution in contractions becomes uniform only at infinity up- and downstream This can be seen from inspection of the hodograph plane as shown in Fig 1 (The sketch is of the logarithmic hodograph plane, $\log(dw/dz) = \log q - i\theta$)

In this plane a uniform flow transforms to a single point, and so the upstream and downstream flows, when uniform, transform, for any boundary shape, to the two points marked

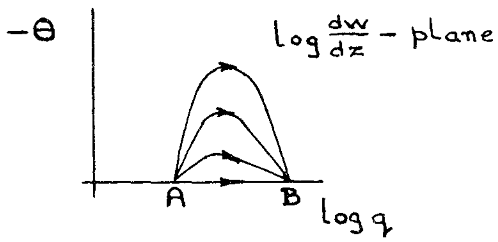

Fig 1

A and B The two uniform flows thus correspond to the origins of a source and sink flow at which φ has the values $-\infty$ and $+\infty$, respectively Transformation back into the z plane by integration[5] gives the corresponding x values as $\mp\infty$

It seems difficult to imagine a flow pattern in the logarithmic hodograph plane where the outer streamline gives a continuous increase in $\log q$, whereas an intermediate one has a maximum and a minimum value The flow along the axis always rises monotonically whatever the boundary shape However, in the real flow a monotonically rising velocity is required only along the boundary

Real contracting ducts are finite in length, usually adjoining parallel wall ducts upstream and downstream As Goldstein showed,[6] two regions of falling velocity gradient occur along the duct boundary There are two general cases:

1) The first case is one in which adverse gradients can be restricted to occur along the parallel walls The boundary in the logarithmic hodograph plane is as sketched in Fig 2, lettered $A\ C\ D\ B$ and marked I, A and B being the origins of the source and the sink flow, respectively

2) The second case is one in which adverse gradients can be restricted to occur along the curved wall Such a boundary is also sketched in Fig 2, lettered $A\ C'\ C\ D\ D'\ B$ and marked II, I, III; cuts occur along AC' and BD' † The resulting curved boundary has three points of inflexion in the z plane, as sketched in Fig 3

One can, of course, specify that condition 1 applies at one end of the duct and condition 2 applies at the other Although the velocity still becomes uniform only in a finite duct at infinity up- and downstream, it approaches its asymptotic value more rapidly than in the infinite length contraction This can be illustrated by considering an approximation to the flow sketched in Fig 2, which is provided by the flow sketched in Fig 4 Here AD and CE are straight lines going to infinity

The flow then consists of a source at A together with its image source; thus, writing the ratio of the velocities at A and B as n, the flow is given by

$$w = \log \log \frac{dw}{dz} + \log\left[\log \frac{dw}{dz} - 2 \log n\right]$$

When $\theta = 0$, $\sin\psi = 0$ and then, for $0 \leq \log q \leq +\infty$,

$$0 < e^{\varphi} \cos\psi \leq +\infty$$

and for $-\log n \leq \log q \leq 0$,

$$-(\log n)^2 \leq e^{\varphi} \cos\psi \leq 0$$

Thus, putting $\psi = 0$ along $A\ D$ gives $\psi = \pi$ along $A\ C$ and also along $A\ D$:

$$e^{\varphi} = \log q \log n^2 q$$

and along $A\ C$

$$e^{\varphi} = -\log q \log n^2 q$$

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† These cuts can extend from A partly toward C and from B partly toward D

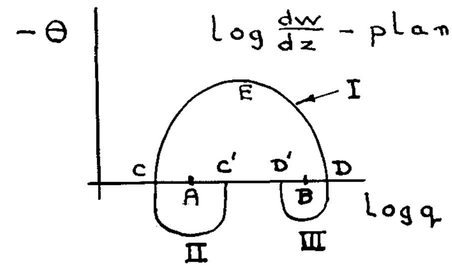

Fig 2

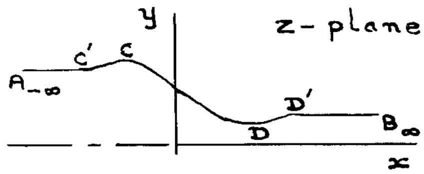

Fig 3

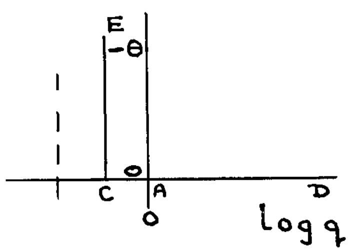

Fig 4

where the right-hand side of these relations can be expanded to give

$$\log q \log n^2 q = 2(\log n)(q - 1) + (1 - \log n)(q - 1)^2 +$$

at the ends of the contraction where the flow is closely uniform, and then $x \propto \varphi \times$ const; thus, the foregoing relations show that in a finite length contraction the velocity along the walls and the centerline approaches its asymptotic values with equal rapidity Along an infinite length contraction for which $n = 1$, the approach of the centerline velocity to the asymptote is comparatively slow; and if one thinks of the length of a contraction as being that distance between two

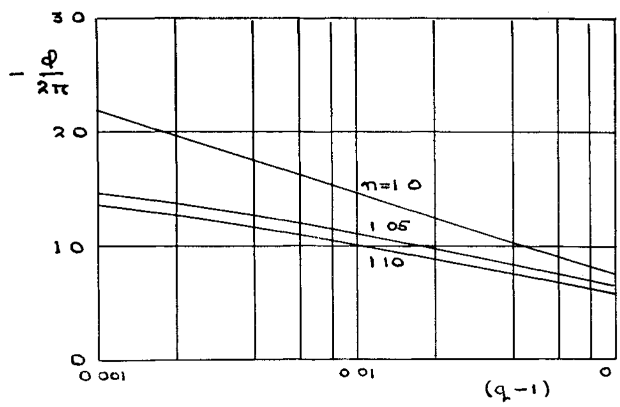

Fig 5

flows of a specified high degree of uniformity, then the effect is large, as is illustrated in Fig 5 At the low-speed end, where $q \simeq 1$, then x is closely proportionate to φ, and the width is approximately 2π If one specifies a velocity distribution that is uniform to 0 1%, then, by using an infinite-length boundary shape, one adds a length of about three-quarters widths to the low speed end The significance of this is brought out when one considers a previous example[5] for a contraction ratio of 2 58 where the finite-length contraction shape had a length of 1 28 times the width at the low-speed end

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